ENGAGING WITH COGNITIVE LEVELS: A PRACTICAL APPROACH TOWARDS ASSESSING THE COGNITIVE SPECTRUM IN MATHEMATICS

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In the interest of further progression beyond the Intermediate Phase, conceptual mastery of various crucial mathematical ideas is necessary. If teaching, learning and assessment remain on the factual recall and operational efficiency levels, progression to the more advanced levels of mathematical activity is severely restricted. In this paper we advocate the integrated assessment of conceptual understanding, across the dimensions of mathematical understanding and on different cognitive levels. We propose a framework for teachers to use in constructing classroom assessments that accommodate the dimensions of understanding at various cognitive levels, to prepare learners for the Senior Phase.

FOCUS AND SIGNIFICANCE OF THE PAPER

This paper is the result of du Plooy's research quest for a practical and reliable way to comply with the CAPS requirement to assess mathematics at different cognitive levels. The study is supervised by Long. In this study some principles informing mathematics teaching and learning design which use specific metacognitive strategies in division at the Intermediate Phase (Grade 6) are investigated.

What emerged while developing the research instruments for the research design was the necessity to enter into the intervention phase with a clear picture of the participants' competency. Firstly, a baseline assessment had to be set enabling a diagnosis of the existing voids in the participants' understanding of the multiplicative structures in real life situations. Secondly, a pre-intervention assessment and a post-intervention assessment had to be set, if the effect of using the proposed metacognitive strategies was to be investigated.

The insights in this paper originated from informal action research in classroom practice and were theoretically grounded through a formal literature review. The practical application of this research finding, culminating in a framework, is that teachers can use this framework to set and evaluate their own assessments according to the requirements of the Intermediate Phase curriculum, for baseline, diagnostic, formative and summative assessment, but particularly to understand how assessment of cognitive levels is done practically within a real classroom setting.

THEORETICAL ASPECTS OF THE FRAMEWORK

A number of theoretical and practical aspects were taken into account while designing the assessments for the study:

- The South African mathematics curriculum. The Curriculum and Assessment Policy Statement (CAPS) (Department of Basic Education (DBE), 2012) was the point of departure in the study;
- The CAPS (DBE, 2012) spells out affective and cognitive goals for mathematics learning. This research is situated in the cognitive domain of mathematics teaching and learning and therefore had to take into account the curriculum requirement of ascribing cognitive levels in assessment (DBE , 2012, p. 296);
- The theoretical antecedents for the cognitive levels used in CAPS were studied in order to apply them in setting the assessments. Similarities were found between the approach in CAPS and the revised form of Bloom's taxonomy for learning, teaching and assessing (Anderson & Krathwohl, 2001; Anderson, 2002). In the revised taxonomy, Anderson and Krathwohl (2001) describe Bloom et al.'s original categories (factual knowledge, conceptual knowledge and procedural knowledge) in action words, and add the metacognitive category of knowledge to the original three categories;
- Mathematics is a complex subject which requires an intricate process of teaching and learning, and multidimensional understandings (Usiskin, 2012) such as understanding the various mathematical representations, the properties of mathematical concepts, the application of operations within certain problems and the understanding of algorithms or methods. Assessment would therefore need to be articulated according to these dimensions of understanding, in addition to the levels of understanding proposed by Anderson and Krathwohl (2001).

THE RELATED LITERATURE

For the purpose of this paper, we focus primarily on the literature regarding cognitive levels of mathematics understanding with a secondary focus on a single selected theory of mathematics understanding, as follows:

Cognitive levels of mathematics understanding

Apart from the different mathematics areas, the CAPS (DBE, 2012, p. 296) also describe four cognitive levels at which assessment has to be conducted. These levels are: knowledge (25%), routine procedures (45%), complex procedures (20%) and problem solving (10%). The four cognitive levels used in CAPS, correspond directly with the Subject Assessment Guidelines of the 1999 TIMSS taxonomy of categories of mathematical demand (Stols, 2013, p. 13). The cognitive categories used in this international test need to be interpreted carefully if they are to be applied in classroom assessment.

For the mathematics teachers, as the users of CAPS, these categories leave room for individual interpretation when an assessment is set. We therefore identified a need for clarity about the concept of cognitive levels as it is explained in the literature.

Linn (2002, pp. 28-37) uses the term "level of cognitive demand" for what CAPS terms "cognitive levels". He discusses the approach taken by the International Assessment of Educational Progress (IAEP), using the three cognitive levels conceptual understanding, procedural knowledge and problem solving. He points out the similarities between the IAEP approach and the five cognitive process categories called performance expectations in TIMSS 1995, namely understanding, routine procedures, and problem solving, investigating and communicating.

It was however Anderson and Krathwohl's (2001) revision of Bloom's taxonomy of educational objectives, that helped the first author to conceptually delineate cognitive levels in assessment. Of the four cognitive categories, namely factual knowledge, procedural knowledge, conceptual knowledge and metacognitive knowledge, the first three have been used in the assessment matrix in Table 1 (Dimensions of understanding and levels of understanding, du Plooy, 2014). The term "factual recall" is used for the first level, meaning "to bring to the fore, knowledge aspects that have been stored in the learners' memories". For the second level "procedural efficiency" is used, adopting Hiebert and Carpenter's (1992) description of procedural knowledge as "a sequence of actions…the manipulation of written symbols in a step-by-step sequence" (p. 78). The third level is termed "*conceptual* grasp". This use of the term is in direct contrast with Feuerstein's (Feuerstein & Rand, 1974, Feuerstein et al, 2006) construct of "*episodic* grasp of reality", where the latter refers to the incidental and isolated experience of reality, unrelated in time and space to other experiences. The construct "conceptual grasp" as used in this study denotes the interrelatedness of an idea with other ideas to form a coherent mental unit, which can be conceptualized as one concept, but large enough to contain different sub-concepts. "Rate" as a mathematical concept is a good example, with speed, unit price, population density and so on, as sub-concepts (see par 3.2).

Usiskin's theory of mathematical understanding

Mathematical activity, according to Usiskin (2012, pp. 2-3, 19) consists of concepts and problems. Within a mathematical problem, several concepts would come to the fore, as is demonstrated in this paper. Usiskin explains that a concept can be used as the unit of analysis, and it is of such a nature that it can be analysed systematically (Usiskin, 2012, p. 15). Usiskin's perspective applies to the research of a concept, and when I interpret it from a teaching point of view, a concept may then be seen as a mathematical unit that can be taught and learned systematically.

Furthermore, Usiskin distinguishes five dimensions[1] (Usiskin. 2012) that constitute mathematics understanding, of which I use four in my assessment matrix of understanding in Table 1 du Plooy, 2014), namely the skill-algorithm understanding, the property-proof understanding, the use-application understanding and the representation-metaphor understanding. In this paper I use this distinction as described by Usiskin (2012, pp. 4-9), as follows:

- The skill-algorithm dimension or procedural understanding essentially entails knowing *how* to get an answer.
- The property-proof dimension involves the identification of the mathematical properties that underlie *why* the particular method of obtaining the answer worked.
- The use-application dimension implies knowing *when* to apply a specific operation.
- The representation-metaphor dimension requires the ability to communicate a mathematical concept by means of an appropriate representation.

The importance of mathematics learning on all cognitive levels

Pantziara and Philippou (2011) draw upon existing studies to make a broad distinction between procedural and conceptual knowledge of mathematics (pp. 61-83). Their research reveals that learners have a better command of some of the problematic mathematical areas like fractions if conceptual knowledge has been developed alongside procedural knowledge. Voutsina (2011, p. 196) describes the relationship between procedural- and conceptual knowledge, and problem solving skills as an iterative process. She found that changes in young children's problem solving behaviour stem from the dynamic interaction between procedural and conceptual knowledge.

Inferring from the literature and du Plooy's own experience it seems entirely plausible that failure to make the cognitive transition from knowledge and routine procedures to complex procedures and problem solving, may account for deteriorating mathematic competence at the more senior levels.

Although factual knowledge and procedural efficiency are important constituents of mathematics proficiency, the transition to the more advanced cognitive levels does not automatically follow on the direct recall of mathematical facts or even on computational efficiency (arriving at the correct answer) in mathematical procedures. This transition rather hinges on true and multidimensional understanding of mathematics concepts. The teaching, learning and assessment processes relating to the conceptual grasp of mathematical constructs are therefore crucial for mathematics progression.

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[1] The fifth dimension, the cultural-historical dimension, is omitted, as it is not relevant in the context of this study.

Subsequently, mathematics assessments have to be created in such a way that they provide for the demonstration of conceptual grasp in addition to factual recall and operational efficiency.

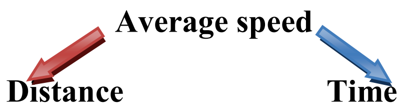
AN ILLUSTRATION OF THE METHOD USED IN THIS PAPER

In setting the assessments it was useful to evaluate test items against the four dimensions of understanding, thus ensuring representivity across the spectrum of mathematics understanding. Furthermore, in the practice of doing this, the need was soon experienced to express, within a specific dimension of understanding, the level of understanding needed to succeed in solving a mathematical problem.

A matrix in Table1(du Plooy, 2014). was subsequently developed according to which test items could be plotted, not only in terms of its dimension of mathematical understanding, but also on the cognitive level that the problem requires. In this matrix Usiskin's (2012) four selected dimensions of understanding are juxtaposed with the first three cognitive levels of Anderson and Krathwohl(2001) as explained above, resembling the *IAEP Mathematics Framework for 9- and 13-Year-Olds* (Linn, 2002, p.35). Anderson and Krathwohl's metacognitive level is not used, since it is not used in the CAPS. The use of this matrix is demonstrated on a test item based on a real-life problem as it would be posed to a Grade 6 learner, as follows:

"Ms Buti drove from Bela-Bela to Bethulie. She started from Bela-Bela at 9:30 am, with the odometer of her car on 88 888 km. She arrived in Bethulie at 16:30, with the odometer on 89 525 km. What was the average speed of Ms Buti's journey?"

The main concepts involved in the above problem can be set out as follows:



"Analogue time" and "digital time" are additional concepts without which the problem cannot be solved, because the problem statement makes use of both.

In the example above, we question whether the factual recall of the term "average speed" on its own can ensure the successful solution of the problem. Here much more understanding is required. We now analyse the above mathematics problem according to the dimensions of understanding (Usiskin, 2012), relating to each concept to the level of understanding that it requires. Broad guidelines are given below as an indication of some possible plotting of concepts onto the matrix, but through an interactive session we shall enrich this list and amend it with more content on each level:

The concept "average speed" requires multidimensional mathematical understanding on different cognitive levels:

a. To understand the meaning of "average speed" and the elements it entails
    Cognitive level: factual recall
    Mathematical dimension: representation-metaphor
b. To understand the elements that "average speed" entails
    Cognitive level: factual recall
    Mathematical dimension: property-proof
c. To understand which operation to use to calculate the average speed
    Cognitive level: factual recall, conceptual grasp
    Mathematical dimension: use-application
d. To understand how to calculate "average speed"
    Cognitive level: operational efficiency
    Mathematical dimension: skill-algorithm

The concept "distance" needs multidimensional understanding:

a. To understand the role of "distance" in calculating average speed
    Cognitive level: factual recall
    Mathematical dimension: representation-metaphor
b. To understand which operation to use in calculating the distance
    Cognitive level: conceptual grasp
    Mathematical dimension: use-application
c. To understand how to calculate "distance" correctly
    Cognitive level: operational efficiency
    Mathematical dimension: skill-algorithm

The concept "time" needs multidimensional understanding:

a. To understand the role of "time" in calculating average speed
    Cognitive level: factual recall
    Mathematical dimension: representation-metaphor
b. To understand which operation to use in calculating the time using the available information
    Cognitive level: conceptual grasp
    Mathematical dimension: use-application
c. To understand how to calculate "time" correctly
    Cognitive level: operational efficiency
    Mathematical dimension: skill-algorithm

Following such an analysis, we can, by way of a practical exercise with participants, map the above mathematics problem on a matrix according to the dimensions of understanding relating to each concept (Usiskin, 2012), at the level of understanding that it requires, as follows:

	Dimensions of understanding (Usiskin, 2012)			
	Use / application	**Skill / algorithm**	**Representation / metaphor**	**Property / proof**
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	Factual recall (the **rule** applicable to the calculation of average speed)	Factual recall (eg **steps** in the division calculation)	Factual recall (the **meaning** of average speed)	Factual recall (**attributes** of average speed)
	Operational efficiency	Operational efficiency (**calculating** towards a correct answer)	Operational efficiency (**formulating** an answer km/h)	Operational efficiency
	Conceptual grasp (this **situation** requires division)	Conceptual grasp	Conceptual grasp (e.g. what **rate** entails)	Conceptual grasp

Table 1: Dimensions of understanding and levels of understanding (du Plooy, 2014 ©)

The above matrix is proposed as a tool in ensuring the representivity of an assessment item across the spectrum of mathematics understanding, on all cognitive levels required for thorough understanding of a concept. It has the potential of extending the present context to a more inclusive concept like "rate". In fact, the transition to the Senior Phase and its increased emphasis on more abstract concepts, make the conceptual grasp of the idea of "rate" an absolute necessity.

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